148. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Helmholtz's differential equation for the strength of an electric current C at any time t, is $C=E/R-L/R\times dC/dt$. Solve this equation, supposing $C=0$ when $t=0$; and E, R, L are to be regarded as constants.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and J. SCHEFFER, A. M. Hagerstown, Md.

The equation can be written $LdC=(E-CR)dt$.

$$\therefore \frac{LdC}{E-CR}=dt, \text{ or } \frac{L}{R}\log(E-CR)+t+B=0.$$

When $C=0$, $t=0$, and $B=-\frac{L}{R}\log E$.

$$\therefore t+\frac{L}{R}\log(E-CR)=\frac{L}{R}\log E, \text{ or } t+\frac{L}{R}\log\left(\frac{E-CR}{E}\right)=0.$$

149. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the volume contained between the plane $z=(a-x)\cot\beta$ and the surface $xz^2=(a-x)(x^2-y^2)$.

Solution by the PROPOSER.

The z limits are $z=\sqrt{\frac{a-x}{x}(x^2+y^2)}$ to $z=(a-x)\cot\beta$.

Eliminating z, $y=\pm\sqrt{[x(a\cot^2\beta-x\operatorname{cosec}^2\beta)]}=y'$.
The x limits are 0 and $a\cos^2\beta=x'$.

$$V=2\int_0^{x'}\int_0^{y'}\left[(a-x)\cot\beta-\sqrt{\frac{a-x}{x}(x^2+y^2)}\right]dxdy$$

$$=\int_0^{x'}\Big[\sqrt{x}\,(c-x)\cot\beta\sqrt{[a\cot^2\beta-x\operatorname{cosec}^2\beta]}$$

$$-x\sqrt{(ax-x^2)}\log\left(\frac{\sqrt{(a-v)}\cot\beta+\sqrt{(a\cot^2\beta-x\operatorname{cosec}^2\beta)}}{\sqrt{x}}\right)\Big]dx.$$

Let $x=a\cos^2\beta\sin^2\theta$.

$$\therefore\ V=2a^3\cos^3\beta\cot^2\beta\int_0^{\frac{1}{2}\pi}(1-\cos^2\beta\sin^2\theta)\sin^2\theta\cos^2\theta d\theta$$

$$-2a^3\cos^5\beta\int_0^{\frac{1}{2}\pi}\sqrt{(1-\cos^2\beta\sin^2\theta)}\sin^4\theta\cos\theta\log\left(\frac{\sqrt{(1-\cos^2\beta\sin^2\theta)}+\cos\theta}{\sin\beta\sin\theta}\right)d\theta.$$

Let $\cos\beta\sin\theta=\sin\varphi$; then the second term becomes

$$2a^3\int_0^{\frac{1}{2}\pi-\beta}\sin^4\varphi\cos^2\varphi\log\left[\frac{\cos\varphi\cos\beta+\sqrt{(\cos^2\beta-\sin^2\varphi)}}{\sin\beta\sin\varphi}\right]d\varphi=2a^3\int_0^{\frac{1}{2}\pi-\beta}(\tfrac{1}{16}\varphi-$$

$$\tfrac{1}{16}\sin\varphi\cos\varphi-\tfrac{1}{24}\sin^3\varphi\cos\varphi+\tfrac{1}{6}\sin^5\varphi\cos\varphi)\frac{\cos\beta d\varphi}{\sin\varphi\sqrt{(\cos^2\beta-\sin^2\varphi)}}.$$

$$\therefore\ V=\tfrac{1}{16}\pi a^3\cos^3\beta\cot^2\beta(1+\sin^2\beta)+\tfrac{1}{8}a^3\cos\beta\int_0^{\frac{1}{2}\pi}d\theta+\tfrac{1}{12}\cos^3\beta\int_0^{\frac{1}{2}\pi}\sin^2\theta d\theta$$

$$-\tfrac{1}{3}a^3\cos^5\beta\int_0^{\frac{1}{2}\pi}\sin^4\theta d\theta-\tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}.$$

$$\therefore\ V=\tfrac{1}{16}\pi a^3\cot\beta\operatorname{cosec}\beta-\tfrac{1}{24}\pi a^3\cos^3\beta$$

$$-\tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}.\quad \tfrac{1}{8}a^3\int_0^{\frac{1}{2}\pi}\frac{\sin^{-1}(\cos\beta\sin\theta)d\theta}{\sin\theta\sqrt{(1-\cos^2\beta\sin^2\theta)}}$$

$$=\tfrac{1}{8}a^3\cos\beta\int_0^{\frac{1}{2}\pi}(1+\tfrac{2}{3}\cos^2\beta\sin^2\theta+\tfrac{8}{15}\cos^4\beta\sin^4\theta+\tfrac{16}{35}\cos^6\beta\sin^6\theta+\ldots.)d\theta$$

$$=\tfrac{1}{16}\pi a^3(\cos\beta+\tfrac{1}{3}\cos^3\beta+\tfrac{1}{5}\cos^5\beta+\tfrac{1}{7}\cos^7\beta+\ldots.)=\tfrac{1}{16}\pi a^3\log\cot\tfrac{1}{2}\beta.$$

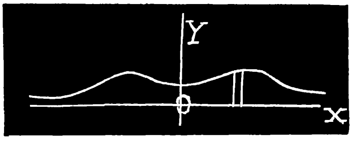
$$\therefore\ V=\tfrac{1}{48}\pi a^3(3\cot\beta\operatorname{cosec}\beta-2\cos^3\beta-3\log\cot\tfrac{1}{2}\beta).$$

150. Proposed by E. B. ESCOTT, Instructor in Mathematics, University of Michigan, Ann Arbor, Mich.

Find total area between the curve $x^4y-x^2+4y-1=0$ and the x-axis.

Solution by J. E. SANDERS, Hackney, Ohio, and the PROPOSER.

The equation may be written $y=\dfrac{x^2+1}{x^4+4}$.



$$\text{Area}=2\int_0^\infty ydx=2\int_0^\infty\frac{x^2+1}{x^4+4}dx$$

$$=\tfrac{1}{4}\left[\int_0^\infty\frac{x+2}{x^2-2x+2}dx-\int_0^\infty\frac{x-2}{x^2+2x+2}dx\right]=\tfrac{1}{8}\Big[\log(x^2-2x+2)$$

$$-\log(x^2+2x+2)+6[\tan^{-1}(x+1)+\tan^{-1}(x-1)]\Big]_0^\infty$$

$$=\tfrac{1}{8}\left[\log\frac{x^2-2x+2}{x^2+2x+2}+6\tan^{-1}\frac{2x}{2-x^2}\right]_0^\infty=\tfrac{1}{8}(6\pi)=\tfrac{3}{4}\pi.\quad \text{Answer.}$$